Math 8301, Manifolds and Topology
Homework 3
Due in-class on **Monday, Oct 1**

1. Using the classification of closed, connected surfaces according to orientability and Euler characteristic, describe the two surfaces obtained by using the following strings to identify edges:
   - $acca^{-1}bdbe^{-1}d^{-1}e^{-1}$
   - $abcb^{-1}defghg^{-1}f^{-1}a^{-1}h^{-1}d^{-1}c^{-1}e^{-1}$

2. An *n-dimensional manifold with boundary* is a topological space M such that every point in M has an open neighborhood U which is homeomorphic to either $\mathbb{R}^n$ or $[0,\infty) \times \mathbb{R}^{n-1}$.

   Assume without proof that no point of $\mathbb{R}^n$ has an open neighborhood homeomorphic to $[0,\infty) \times \mathbb{R}^{n-1}$. Define the *boundary* ∂M of an n-dimensional manifold with boundary, show that it is an $(n-1)$-dimensional manifold, and show that if M is compact then ∂M is closed.

3. An *n-dimensional manifold with corners* is a topological space M such that every point in M has an open neighborhood U which is homeomorphic to $[0,\infty)^p \times \mathbb{R}^{n-p}$ for some $0 \leq p \leq n$.

   Show that any n-dimensional manifold with corners is an n-dimensional manifold with boundary.

4. Suppose that a topological space X has a function $m : X \times X \to X$. Show that if α and β are *any* paths in X, the definition

   $$(\alpha * \beta)(t) = m(\alpha(t), \beta(t))$$

   is homotopy invariant, in the sense that $[\alpha] * [\beta] = [\alpha * \beta]$ is well-defined on homotopy classes of paths.

5. Show that the product of the previous problem satisfies an interchange law

   $$(\alpha \cdot \beta) * (\gamma \cdot \delta) = (\alpha * \gamma) \cdot (\beta * \delta)$$

   whenever the left-hand side is defined.