Math 8302, Manifolds and Topology II

Final exam

Due May 17, 2013 by 3pm (return to my box in the mailroom)

1. Suppose X is a connected, smooth manifold with basepoint x and Y is its universal cover, which inherits the structure of a smooth manifold together with an action of $\pi_1(X, x)$ by smooth maps. Show that, for any n, the projection map $\pi : Y \to X$ gives rise to an isomorphism

$$\pi^* : \Omega^n(X) \to [\Omega^n(Y)]^{\pi_1(X,x)}$$

between differential forms on X and differential forms on Y which are invariant under the action of $\pi_1(X, x)$.

2. Show that the wedge product $\wedge$ of differential forms induces a well-defined map $H^p_{dR}(M) \times H^q_{dR}(M) \to H^{p+q}_{dR}(M)$ which is associative and distributive over addition, making the de Rham cohomology into a ring.