Math 8302, Manifolds and Topology II
Homework 2
Due in-class on **Wednesday, February 13**

1. Give a definition of the complex Grassmannian $Gr_{\mathbb{C}}(n, m)$ of n-dimensional subspaces of $\mathbb{C}^m$, and show that it is a smooth manifold.

2. Suppose K and L are closed subsets of a smooth manifold M, and $f, g : M \to \mathbb{R}$ are smooth maps which are equal on an open set containing $K \cap L$. Show that there is a smooth map $h : M \to \mathbb{R}$ such that $h|_K = f|_K$ and $h|_L \equiv g|_L$.

3. Suppose M is a compact manifold. Show that there is a smooth map $M \to \mathbb{R}^N$ for some large N which is injective.

4. Consider the point $(3, 4, 1)$ in Cartesian coordinates on $\mathbb{R}^3$. Give a formula for how a tangent vector (a, b, c) at this point is translated into cylindrical coordinates (r, θ, z), where $(x, y, z) = (r\cos(\theta), r\sin(\theta), z)$.

5. Show explicitly, using the abstract definition of a tangent vector, that the set of tangent vectors at a point p forms a vector space whose dimension equals the dimension of the manifold.