Math 8306, Algebraic Topology
Homework 12
Due in-class on **Wednesday, December 3**

1. Show that if a principal bundle $P \to B$ has a section, then there is a homeomorphism to the trivial principal bundle: $P \cong B \times G$ as right $G$-spaces.

2. Let $G$ and $H$ be topological groups. Suppose $P_1 \to B$ is a principal $G$-bundle and $P_2 \to B$ is a principal $H$-bundle. Show $P_1 \times_B P_2$ (pullback!) is a principal $G \times H$-bundle.

3. Suppose $G \to H$ is a homomorphism of topological groups, and $P \to B$ is a principal $G$-bundle. Show that the mixing construction gives a *principal* $H$-bundle $P \times_G H \to B$.

4. We identified $\mathbb{CP}^1$ with the space of lines in $\mathbb{C}^2$. Associated to this, there is a vector bundle $\xi \to \mathbb{CP}^1$:

$$\xi = \{(L, v) | L \in \mathbb{CP}^1, v \in L\}.$$

   Find an open cover $\{U_\alpha\}$ together with transition functions $\{h_{\alpha,\beta} : U_\alpha \cap U_\beta \to GL_1(\mathbb{C})\}$ to reconstruct the associated principal $GL_1(\mathbb{C})$-bundle.