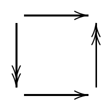
Math 8306, Algebraic Topology
Homework 8
Due in-class on **Monday, October 27**

1. Let X be a Klein bottle:

   We can put a Δ-complex structure on X with one vertex p, three edges a, b, c, and two 2-simplices u, v. Make this Δ-complex structure explicit, and use it to compute $H^*(X; \mathbb{Z}/2)$ together with the cup product on it.

2. In Hatcher, pg. 131, exercise 8, there is given a description of a *lens space* formed by gluing together n tetrahedra; let's call this $L(n, 1)$. (The 1 is because we are gluing the "bottom" face of T_i to the top face of T_{i+1}.) Compute $H^*(L(n, 1); \mathbb{Z}/n)$ together with the cup product on it.

3. We know that if X and Y are based spaces, the wedge $X \vee Y$ has

   $$H^k(X \vee Y; R) = H^k(X; R) \oplus H^k(Y; R)$$

   for any $k > 0$. Show that under this identification, the cup product is given by

   $$(\alpha, \beta) \cup (\alpha', \beta') = (\alpha \cup \alpha', \beta \cup \beta').$$