Math 8307, Algebraic Topology II

Homework 3

Due in-class on **Wednesday, February 11**

1. Show that fibrations are closed under retracts: If there is a diagram

$$\begin{array}{ccccc} A & \longrightarrow & X & \longrightarrow & A \\ \downarrow & & \downarrow & & \downarrow \\ B & \longrightarrow & Y & \longrightarrow & B \end{array}$$

   of spaces such that both horizontal composites are identity maps, and $X \to Y$ is a (Serre) fibration, show that $A \to B$ is a (Serre) fibration.

2. Suppose $U \subset X$ is an open subset, and let j be the inclusion map. Show that the projection $p : M_j \to X$ from the mapping cone of j to X is a Serre fibration. (Hint: Use one of the major theorems from point-set topology.)

3. Suppose that $i : A \to X$ is a cofibration, and let M_i be the mapping cylinder. Show that X/A is homotopy equivalent to the mapping cone $M_i/Atimes1$.

4. Suppose that $A \to X$ is a cofibration and $f : A \to Y$ is a map. We can form a new space $X \cup_A Y$ by gluing X to Y along A. Show that the map $Y \to X \cup_A Y$ is a cofibration.