18.704 Problem Set 2

Due Friday, Mar. 3, at **3pm** in 2-171

1. So far in the course, we've only been considering representations over the complex numbers $\mathbb{C}$, but we could equally consider real representations, which are given by homomorphisms from G to $\mathrm{GL}_n(\mathbb{R})$. What part of the proof of Schur's Lemma (Proposition 4) fails to be true for real representations? Give an example where it fails.

2. (TeX question) Typeset the statement and proof of Proposition 3 into TeX. (It doesn't need to be *exactly* the same.) Submit printouts of both your TeX code and the resulting output. On the course website you'll find a link to a file called "example1.tex" that will give you some basic instructions if you've never used it before.

3. If χ and λ are functions on a group G, we define their *convolution* $\chi * \lambda$ by the formula
$$(\chi * \lambda)(g) = \frac{1}{|G|} \sum_{x \in G} \chi(gx^{-1})\lambda(x).$$
Show that if both of χ and λ are class functions on G, so is $\chi * \lambda$.

* (Bonus, not mandatory) Use the orthogonality relations for matrix coefficients from section 2.2 to find a formula for the convolution when χ and λ are the characters of irreducible representations.