18.704 Problem Set 3

Due Friday, Mar. 17, at **3pm** in 2-171

At least one of your answers must be typeset in TeX.

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1. Serre, Exercise 3.3.

2. Compute the character table of the following group of order 20.

$$\langle x, y \mid x^5 = y^4 = e, yxy^{-1} = x^2 \rangle$$

3. Suppose a group G acts on a set X, and $|X| > 1$. We say that this action is *doubly transitive* if for any two pairs (x, y), (x', y') of points of X such that $x \neq y$ and $x' \neq y'$, there is an element $g \in G$ such that $(gx, gy) = (x', y')$. Show that if G acts doubly transitively on X with character χ, then

$$\begin{aligned} \langle \chi, \chi \rangle &= 2, \text{ and} \\ \langle \chi, 1 \rangle &= 1, \end{aligned}$$

where 1 is the trivial character. What does this tell you about the associated representation?