18.905 Problem Set 4

Due Wednesday, October 4 in class

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1. Let A be the 1-skeleton of Δ^3, i.e. the union of the vertices and lines. Let CA, the cone on A, be the union of all the lines joining A to the center of Δ^3. Compute the homology groups $H_*(CA, A)$.

2. Hatcher, exercise 17 on page 132.

3. Hatcher, exercise 27 on page 133.

4. A *generalized homology theory* E is a collection of functors E_n from the category of pairs (X, A) to the category of abelian groups, together with natural transformations $\partial_E : E_n(X, A) \to E_{n-1}(A, \emptyset)$ which satisfy the Eilenberg-Steenrod axioms except possibly for the dimension axiom. Homology H is one example. In the previous problem set we showed that H^Y, given by functors $H_n^Y(X, A) = H_n(X \times Y, A \times Y)$, is a generalized homology theory.

   A *map* of generalized homology theories $\phi : E \to F$ is a collection of natural transformations $\phi_n : E_n \to F_n$ such that $\partial_F \circ \phi_n = \phi_{n-1} \circ \partial_E$.

   Show that, with this definition, generalized homology theories form a category. Given a map $f : Y \to Z$ of spaces, explain why we get a map of generalized homology theories $H^f : H^Y \to H^Z$. Show that this is a functor from spaces to generalized homology theories.