18.905 Problem Set 6

Due Wednesday, October 18 in class

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1. Use the universal coefficient theorem to compute $H_*(L(p,q);\mathbb{Z}/m)$ for all m, where $L(p,q)$ are the lens spaces defined in class.

2. Hatcher, Exercise 1 on page 267.

3. Given an arbitrary finitely generated abelian group M, compute $\mathrm{Tor}_k(\mathbb{Q}/\mathbb{Z}, M)$ for all $k \geq 0$. (Bonus marks for doing an arbitrary abelian group.)

4. The n-dimensional chains C_n form a functor from spaces to abelian groups. Suppose F is another functor from spaces to abelian groups. Show that any natural transformation Θ from C_n to F must be as follows: For any space X, the map $\Theta_X : C_n(X) \to F(X)$ is given by

$$\Theta_X\left(\sum m_\sigma \sigma\right) = \sum m_\sigma F(\sigma)(\Theta_{\Delta^n}\Delta^n).$$

(Here we recall that the chain $\Delta^n \in C_n(\Delta^n)$ is represented by the identity map from Δ^n to itself.) Conversely, given an element $S \in F(\Delta^n)$, show that we can define a natural transformation Θ from C_n to F by

$$\Theta_X\left(\sum m_\sigma \sigma\right) = \sum m_\sigma F(\sigma)(S).$$