18.906 Problem Set 5 (New Version)

Due Wednesday, March 14 in class

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1. Suppose $f : X \to Y$ is a map of 1-connected CW-complexes such that the induced map $f_* : H_*(X) \to H_*(Y)$ is an isomorphism. Show that f is a homotopy equivalence. (Hint: Replace f with a cofibration and identify the first nonvanishing relative homotopy group.)

2. Suppose $X = K(G, n)$ and $Y = K(H, n)$ are based CW-complexes which are Eilenberg-Maclane spaces. Show that the functor π_n gives an isomorphism
$$[X, Y]_* \to \mathrm{Hom}(G, H).$$
(Don't assume that X and Y are necessarily constructed by the same procedure as in class.)

3. The topological group S^1 acts on the unit sphere $S^{2n+1} \subset \mathbb{C}^{n+1}$ via
$$\lambda \cdot (z_0, \cdots, z_n) = (\lambda z_0, \cdots, \lambda z_n)$$
with quotient space $\mathbb{CP}^n$. You may assume that this action has transverse slices. Compute $\pi_k(\mathbb{CP}^n)$ in as large a range as you can.

   Let $\mathbb{CP}^\infty = \cup_n \mathbb{CP}^n$. What are the homotopy groups of $\mathbb{C}P^\infty$?

4. Same question as the previous problem with $\{\pm 1\}$ acting on $S^n \subset \mathbb{R}^{n+1}$ with quotient $\mathbb{RP}^n$, and union $\mathbb{R}P^\infty$.