18.906 Problem Set 9

Due Wednesday, April 18 in class

---

1. We showed in a previous problem set that the map $O(n) \to O(n+1)$ is $(n-1)$-connected. Show that the map $BO(n) \to BO(n+1)$ is n-connected as a result.

   Conclude the following *cancellation theorem:* Suppose X is a d-dimensional CW-complex, $d < n$, with n-dimensional vector bundles ξ_1 and ξ_2. Let ε be the trivial vector bundle on X. Show that if $\xi_1 \oplus \varepsilon \cong \xi_2 \oplus \varepsilon$, we must have $\xi_1 \cong \xi_2$.

2. If X is a finite CW-complex, show that any vector bundle on X is a subbundle of a trivial bundle $\oplus^n \varepsilon$. (Hint: First show it for the canonical bundles on the finite Grassmannians $\mathrm{Gr}(k, n)$.)

3. Suppose that we have nonnegative integers a, b with binary expansion $a = a_n a_{n-1} \ldots a_1 a_0 = \sum a_k 2^k$ and similarly $b = b_n b_{n-1} \ldots b_1 b_0$. (Some of the leading digits might be 0.) Show that we have an identity of binomial coefficients
$$\binom{a}{b} \equiv \prod_{i=0}^{n} \binom{a_i}{b_i} \pmod 2.$$

4. Let γ be the canonical real line bundle on $\mathbb{RP}^n$, classified by the nontrivial element in $H^1(\mathbb{RP}^n; \mathbb{Z}/2)$. Use Stiefel-Whitney classes and the previous problem to show that $\oplus^k \gamma$ cannot possibly be the trivial bundle unless k is a multiple of 2^m, where $2^{m-1} \le n < 2^m$.